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FREE CONVECTION EFFECTS ON THE OSCILLATORY FLOW OF A VISCOUS, INCOMPRESSIBLE FLUID PAST A STEADILY MOVING VERTICAL PLATE WITH CONSTANT SUCTION

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Abstract—An analysis of a two-dimensional flow of an incompressible, viscous fluid past an infinite porous plate is presented under the following conditions: (i) suction velocity normal to the plate is constant; (ii) the free stream velocity oscillates in time about a constant mean; (iii) the plate temperature is constant; (iv) existence of free convection currents due to temperature difference between the plate temperature and the free stream temperature; (v) the plate is moving steadily in the upward or downward direction. On solving the coupled non-linear equations in approximate way, the results for the mean velocity and temperature, the mean skin-friction, the mean rate of heat transfer, the transient velocity and temperature and the amplitude and the phase of the skin-friction and the rate of heat transfer are shown graphically. It is observed that for air, water, the mean skin-friction decreases when the plate, being cooled by free convection currents, is in upward motion and increases when it is in downward motion. For the plate being heated by free convection currents, there is a fall in the mean skin-friction in case of upward motion and a rise in the mean skin-friction for downward motion. The results are presented in the quantitative manner during the course of discussion.

NOMENCLATURE

$\|B\|$,	amplitude of the skin-friction;
c_p,	specific heat at constant pressure;
E,	Eckert number $(U_0^2/c_p(T_w' - T_\infty'))$;
g_x,	acceleration due to gravity;
G,	Grashof number $(\nu g_x \beta(T_w' - T_\infty')/U_0 v_0^2)$;
K,	thermal conductivity;
M_r, M_i,	fluctuating parts of the velocity profile;
P,	Prandtl number $(\mu c_p/K)$;
p,	pressure;
q,	dimensionless rate of heat transfer $[q'\nu/v_0 K(T_w' - T_\infty')]$;
$\|Q\|$,	amplitude of the rate of heat transfer;
t,	dimensionless time $(t'v_0^2/4\nu)$;
T',	temperature of fluid;
T_w',	temperature of the plate;
T_∞',	temperature of the fluid in the free stream;
T_r, T_i,	fluctuating parts of the temperature profile;
u', v',	velocity components in the x', y' directions;
u,	dimensionless velocity (u'/U_0);
v_0,	suction velocity;
U',	free stream velocity;
U_0,	mean of $U'(t')$;
U,	dimensionless free stream velocity (U'/U_0);
U_0,	amplitude of free stream fluctuations;
u_0,	mean steady velocity;
u_1,	unsteady part of the velocity;
x', y',	co-ordinate system;
y,	dimensionless co-ordinate normal to the wall $(y'v_0/\nu)$.

Greek symbols

ω',	frequency of free stream oscillations;
ω,	dimensionless frequency $(4\nu\omega'/v_0^2)$;
τ',	skin-friction;
τ,	dimensionless skin-friction $(\tau'/\rho U_0 v_0)$;
θ,	dimensionless temperature $(T' - T_\infty'/T_w' - T_\infty')$;
θ_0,	mean steady temperature;
$\varepsilon\theta_1$,	amplitude of the temperature fluctuations;
α,	phase angle of the skin-friction;
β,	phase angle of the rate of heat transfer;
β_1,	coefficient of volume expansion;
ρ',	density of fluid in the boundary layer;
ρ_∞',	density of fluid in the free stream;
ν,	kinematic viscosity;
μ,	viscosity.

INTRODUCTION

IN RECENTLY published papers by Soundalgekar [1,2], an analytical study of the free convection currents on the mean flow and the unsteady flow, past an infinite vertical porous plate, when the free stream is oscillating about a non-zero constant mean, was presented. The study of the mean flow being important from technological point of view, it was described separately. The method suggested by Lighthill [3] and used by Stuart [4] was employed to solve the problem completely. In these two papers, the fluids considered were mainly air and water at normal temperature. But as the behaviour of water at 4°C has been observed to be different from that of water at ordinary temperature of 20°C, Soundalgekar [5] discussed the effects of free convection currents on the flow of water at 4°C past an infinite, vertical porous plate in the presence of the free stream oscillating about a non-zero constant mean.

In these investigations, the vertical plate was assumed to be stationary. Now, if the vertical plate is assumed to be moving steadily in the vertical or downward direction with a constant velocity, then how the flow of air or water is affected by the free convection currents or frequency? This is the motivation of this paper.

The assumptions and the mathematical analysis being described in [1, 2] it is only assumed here that the steady motion of the vertical plate is such that the laminar flow of fluid is maintained, i.e. the vertical plate is moving with a velocity which is assumed to be very small. Again due to the motion of the plate, the constant suction velocity which was assumed to be normal to the plate in [1, 2, 5] will be inclined to the normal to the plate. This inclination is assumed to be very small and hence negligible. This is possible if the plate is moving very slowly. In Section 2, the non-dimensional equations governing the flow, as derived in [1] are assumed and the boundary conditions are modified. Following the procedure of (1), the solutions for the mean velocity, the mean temperature, the mean-skin-friction, the mean rate of heat transfer and the transient velocity and temperature, are derived. The results for air and water at normal temperature are discussed in a quantitative way. In Section 3, the main results are summarized.

MATHEMATICAL ANALYSIS

The equations, in non-dimensional form, governing the flow are equations (14), (15) from [1]

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial U}{\partial t}+G\theta+\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t}-P\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2}+PE(\partial u/\partial y)^2. \tag{2}$$

All the physical variables and, the non-dimensional quantities are described in Notation. The boundary conditions are

$$u(0)=\pm V,\ \theta(0)=0;\quad u(\infty)=U(t),\ \theta(\infty)=0 \tag{3}$$

where $V = V_1/U_0$ is the steady velocity of the plate in dimensionless form and the + or − sign denotes the motion in the upward or downward direction.

Following the procedure as described in [1], we now derive the expressions for the velocity and the temperature as follows:

$$u_0(y)=1+A\,e^{-y}-B\,e^{-Py}+GEP\Bigg[\frac{1}{P^2-P}\left\{\frac{A^2}{4-2P}-\frac{2GA}{P^2-1}+\frac{B^2}{2}\right\}(e^{-y}-e^{-Py})-\frac{A^2}{4(2-P)}(e^{-y}-e^{-2y})+\frac{2BA}{(P+1)^2}\times(e^{-y}-e^{-(P+1)y})-\frac{B^2}{2(4P^2-2P)}\times(e^{-y}-e^{-2Py})\Bigg] \tag{4}$$

$$A=B-1+V,\quad B=G/P^2-P$$

$$u_1(y)=1-e^{-my}+PGE\left[\frac{2mA}{\alpha}\left(\frac{e^{-my}-e^{-(m+1)y}}{2m}-\frac{e^{-my}-e^{-ny}}{\gamma}\right)+\frac{2mG}{(P-1)\beta}\left\{\frac{e^{-(m+P)y}-e^{-my}}{P(2m+P-1)}+\frac{e^{-my}-e^{-ny}}{\gamma}\right\}\right] \tag{5}$$

$$\theta_0(y)=e^{-Py}+EP\left[\frac{A^2}{4-2P}(e^{-PY}-e^{-2y})-\frac{2GA}{P^2-1}\times(e^{-PY}-e^{-(P+1)y})+\frac{B^2}{2}(e^{-Py}-e^{-2Py})\right] \tag{6}$$

$$\theta_1(y)=2mPE\left[\frac{A}{\alpha}(e^{-(m+1)y}-e^{-ny})+\frac{BP}{P\beta}(e^{-ny}-e^{-(m+P)y})\right] \tag{7}$$

$$m=\frac{1+\sqrt{(1+i\omega)}}{2},\quad n=\frac{P+\sqrt{(P^2+i\omega P)}}{2},$$

$$\alpha=(m+1)^2-P(m+1)-\frac{i\omega P}{4},$$

$$\beta=m(m+P)-\frac{i\omega P}{4},\quad \gamma=n^2-n-\frac{i\omega}{4}.$$

The mean velocity $u_0(y)$ and the mean temperature $(\theta_0(y)$ are shown on Figs. 1–3 for air ($P = 0{\cdot}71$) and water ($P = 7$). These are the values of the Prandtl number at 20°C. But the behaviour of water at 4°C being different, this case will be discussed in the next paper to follow soon.

DISCUSSION OF PREDICTIONS

The mean velocity is shown on Figs. 1 and 2, in case of cooling and heating of the plate by the free convection currents. Due to upward motion of the plate, there is a rise in the value of the mean velocity in the upward direction. Similarly there is a fall in the value of the mean velocity in case of downward motion. In order to understand the effects of greater cooling of the plate by the free convection currents when the plate is in motion, we give a quantitative estimate of the percentage changes in the maximum value of the mean velocity. Thus, in the case of the stationary plate ($V = 0$), for $E = 0{\cdot}01$, there was 115 per cent increase [1] in the maximum velocity due to greater cooling of the plate such that G is increased from 5 to 10. Under similar circumstances, when $V = 1$, there is observed to be 108 per cent increase in the maximum value of the mean velocity. This leads us to conclude that the rate of increase in the maximum value of the mean velocity is less as compared to that in case of a stationary plate. Similarly for $G = 5$, when the value of E is doubled, there is 5 per cent increase [1] in the maximum mean

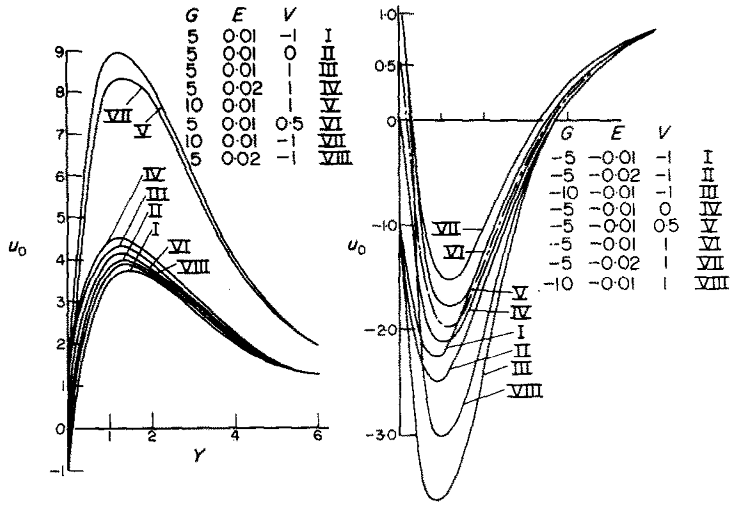

FIG. 1. Mean velocity profiles, $P = 0{\cdot}71$.

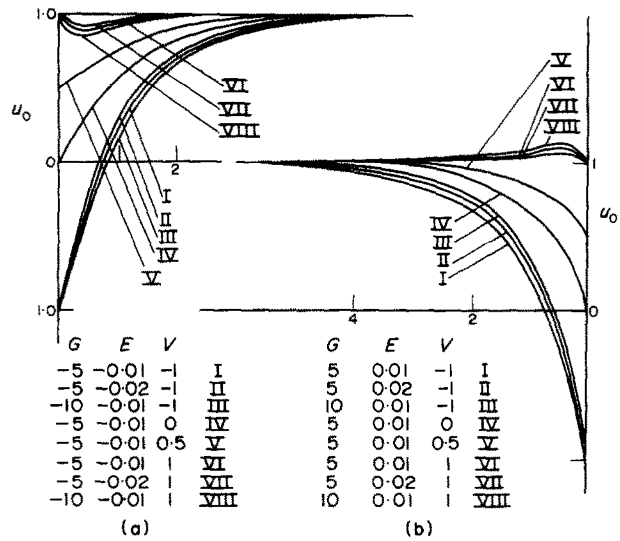

FIG. 2. Mean velocity profiles, $P = T$.

velocity when $V = 0$ and 4·6 per cent increase in the maximum mean velocity when $V = 1$. Thus, due to greater viscous dissipative heat, the rate of increase in the mean velocity falls in case of the plate moving in the upward direction. Again, we observe that in case of heating of the plate by the free convection currents, an upward or downward motion causes a rise or fall in the mean velocity respectively. For $V = 0$, there is 10 per cent [1] increase in the maximum velocity when E is doubled and for $V = 1$, there is 16 per cent increase in the maximum velocity when E is doubled. This leads us to conclude that when the plate is in upward motion, the rate of increase in the maximum mean velocity is more as compared to that in case of stationary plate. Physically, this can be explained as follows: There is more viscous dissipative heat generated due to upward motion of the heated plate which causes the mean velocity to increase rather fast. To understand the effects of greater heating of the plate by the free convection currents, when the plate is moving in the upward direction, in a quantitative way, on the mean velocity of air, we observe that for $V = 0$, there is 66 per cent [1] decrease in the maximum mean velocity and for $V = 1$, there is 70·4 per cent decrease in the maximum mean velocity when G is increased from 5 to 10. This leads us to conclude that due to greater heating of the plate by the free convection currents, the rate of decrease of the maximum mean velocity is more in case of the plate moving in the upward direction than than in case of the stationary plate.

We now study a similar phenomenon in a quantitative manner for the plate ($G \gtrless 0$) moving in the downward direction.

As compared with 115 per cent [1] increase in the maximum mean velocity for $V = 0$ and G increased from 5 to 10, there is 124 per cent increase in the maximum mean velocity under same conditions when $V = -1$. We conclude that due to downward motion

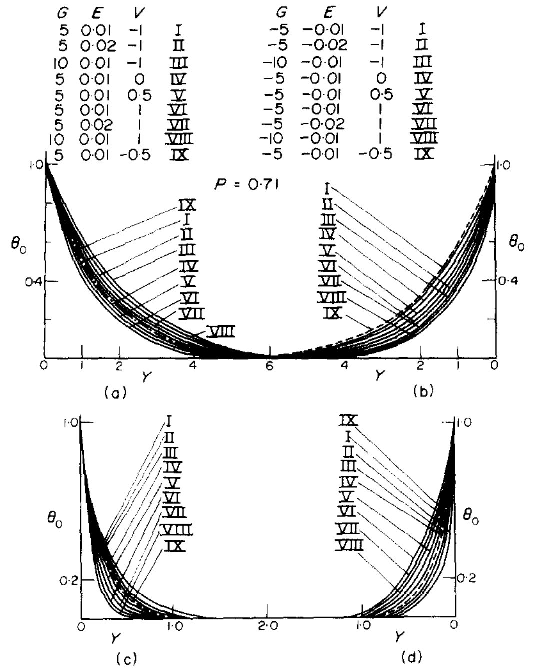

FIG. 3. Mean temperature profiles.

of the plate, the rate of increase in the maximum mean velocity is more owing to greater cooling of the plate. Again for $G > 0$, the rate of increase in the maximum mean velocity is more for downward motion of the plate as compared to that for upward motion of the plate. For $G > 0$ and $V = -1$, when E is doubled, there is 4 per cent increase in the maximum mean velocity. This shows that the rate of increase in the maximum mean velocity, due to greater viscous dissipative heat, falls owing to the downward motion of the plate as compared to the stationary plate or the plate moving in the upward direction.

For $E = 0{\cdot}01$, $V = -1$, there is 69 per cent decrease in the maximum mean velocity when G is changed from -5 to -10 due to greater heating of the plate by the free convection currents as compared to 66 per cent [1] for $V = 0$. Hence the rate of decrease in the maximum mean velocity is more due to downward motion of the heated plate as compared to the stationary plate, but it is less as compared to the upward motion of the heated plate. Again to understand the effects of more addition of the viscous dissipative heat, when the plate is moving in the downward direction, we observe that for $G = -5$, $V = -1$, there is 10·6 per cent decrease when E is doubled. Thus as compared to stationary plate (10 per cent [1]) the rate of decrease in the maximum mean velocity is negligible when $V = -1$.

In case of air, we observe that the mean velocity profiles, for $G < 0$, are observed to be of reverse type for both upward or downward motion of the plate.

We have studied the behaviour of the mean velocity in case of air. We now study the same in case of water from Fig. 2.

It is interesting to note the behaviour of the mean velocity of water past a plate heated by free convection currents and moving in the upward direction (Fig. 2a). Owing to greater viscous dissipative heat, there is a rise in the mean velocity. Quantitatively, for $G = -5$, $V = 1$, there is 4·5 per cent rise in the mean velocity of water at $y = 0{\cdot}4$ when the value of E is doubled. But owing to greater heating of the plate, there is a fall in the mean velocity of water. Thus, for $E = -0{\cdot}01$, $V = 1$, there is 3·4 per cent fall in the mean velocity at $y = 0{\cdot}4$ owing to an increase in G from 5 to 10. The effects of G and E are the same when the plate is moving in the downward direction. This is not the case when the plate is moving in the upward direction and at the same time is being cooled by the free convection currents. Owing to greater cooling of the plate or due to greater viscous dissipative heat, there is a rise in the mean velocity of water near the plate. Thus at $y = 0{\cdot}5$ and $V = 1$, when the value of E is doubled there is 3·7 per cent rise in the mean velocity for $G = 5$ whereas when the value of G is increased from 5 to 10, there is 7·5 per cent rise in the mean velocity of water at $y = 0{\cdot}5$ and $V = 1$, $E = 0{\cdot}01$.

The mean temperature profiles calculated from (6) are shown on Fig. 3 for air and water respectively. The curves in Fig. 3(a) correspond to the plate being cooled by free convection currents. It is interesting to note here that the mean temperature rises due to the downward motion of the plate and falls owing to upward motion of the plate when the values of G and E are constant. However, in case of both upward and downward motion of the plate, owing to greater viscous dissipative heat or due to greater cooling of the plate, there is a rise in the mean temperature. It is also important to note that for constant value of G and E, a further increase in the velocity of the plate in the upward direction leads to a fall in the mean temperature of the air, whereas a further increase in the velocity of the plate in the downward direction leads to an increase in the mean temperature of air.

The curves on Fig. 3(b) correspond to the case of heating of the plate by the free convection currents. We observe here that the mean temperature increases due to the downward motion of the plate and decreases due to upward motion of the plate. In case of downward and upward motion of the plate, owing to greater viscous dissipative heat or due to greater heating of the plate, there is a fall in the mean temperature of air. An increase in the velocity of the plate in the upward or downward direction leads to a fall in the mean temperature of air.

The curves on Fig. 3(c) correspond to the case of cooling of the plate by the free convection currents. The effect of upward or downward motion of the plate on the mean temperature of water is the same as in case of air. The effects of G and E on the mean temperature of water are also the same as in case of air, when there is upward motion of the plate. But in case of downward motion of the plate, owing to greater viscous dissipative heat there is a rise in the mean temperature of water whereas due to greater cooling of the plate, there is a fall in the mean temperature of water. An increase in the plate velocity in the upward direction causes a fall in the mean temperature and that in the downward direction causes a rise in the mean temperature of water. Figure 3(d) shows the mean temperature profiles for water in the presence of the plate being heated by the free convection currents. Owing to downward motion of the plate, there is a fall in the mean temperature of water and it falls still more due to an increase in the plate velocity in the downward direction. But owing to the velocity of the plate in the upward direction, there is a rise in the mean temperature which rises again due to further increase in the velocity of the plate in the upward direction. When the plate is in the downward motion, due to greater viscous dissipative heat or owing to greater heating of the plate, there is a fall in the mean temperature and same are the effects of G and E when the plate is moving in the upward direction.

Knowing the velocity field, it is now proposed to study the behaviour of the skin-friction. Following the procedure of [1], the expression for the mean skin-friction is given by

$$\tau_m = \left.\frac{du_0}{dy}\right|_{y=0}$$
$$= -A+BP+GEP\left[\frac{1}{P}\left(\frac{A^2}{4-2P}\frac{2GA}{P^2-1}+\frac{B^2}{2}\right) - \frac{A^2}{4(2-P)}+\frac{2BAP}{(P+1)^2}-\frac{B^2}{4P}\right]. \quad (8)$$

τ_m is shown on Fig. 4(a, b) for air and water. In case of air, when the plate is being cooled by the free convection currents, τ_m decreases due to the upward motion of the plate and still decreases more than the velocity in the upward direction is increased. But when the plate is moving in the downward direction, the mean skin-friction increases and it increases more due to more increase in the velocity in the downward direction. For experimental verification, we observe that for $G = 5$, $E = 0{\cdot}01$, there is 12·1 per cent decrease in the value of the mean skin-friction when the velocity of the plate in the upward direction is raised from 0 to 1. Under similar circumstances for downward motion of the plate, there is 12·4 per cent rise in the value of the mean skin-friction when V is changed from 0 to 1. Greater cooling of the plate causes also a rise in the mean skin-friction. Quantitatively we observe that for $E = 0{\cdot}01$, when G is increased from 5 to 10, there is 117 per cent [1] rise in the mean skin-friction when $V = 0$ and 134 per cent rise in the mean skin-friction when $V = 1$. This leads us to conclude that there is a rise in the rate of increase in the mean skin-friction owing to greater cooling of the plate moving in the upward direction. Under similar circumstances, when the plate is moving downward, $V = -1$, there is 106 per cent rise in the mean skin-friction. Thus, owing to greater cooling of the plate, the rate of increase in the mean skin-friction is more when the plate is moving in the upward direction than that when it is moving in the downward direction.

When the plate is being cooled by the free convection currents, owing to greater viscous dissipative heat, there is also a rise in the mean skin-friction. Thus for $G = 10$, when E is increased from 0·01 to 0·02, there is 22 per cent rise in the mean skin-friction when $V = 0$, 17 per cent rise when $V = 1$ and 21 per cent rise when $V = -1$. Hence, viscous dissipative heat causes more rise in the value of τ_m in case of downward motion of the plate.

We now study the behaviour of the mean skin-friction in water when the plate is being cooled by the free convection currents. For $G = 5$, $E = 0{\cdot}01$, there is 58·7 per cent fall in the value of the mean skin-friction when the plate is moving in the upward direction such that V is changed from 0 to 1. Under similar circumstances, for downward motion, where V is changed from 0 to -1, there is 60·4 per cent rise in the value of the mean skin-friction. Thus, even if the value of the mean skin-friction in the case of water is less as compared to that of air, due to an upward or downward motion of the plate in water, the rate of decrease or increase in the mean skin-friction is more in case of water. Owing to greater cooling of the plate by the free convection currents, there is always a rise in the mean skin-friction. Quantitatively, for $E = 0{\cdot}01$ and G increased from 5 to 10, there is 41·8 per cent rise in the mean skin-friction for $V = 0$, 100 per cent rise for $V = 1$ and 25 per cent rise for $V = -1$. Thus as compared to air, the percentage rise in the mean skin-friction owing to greater cooling of the plate is less in case of water.

When the plate is being heated by the free convection currents, for air, we observe that due to upward motion of the plate, there is a fall in the mean skin-friction and owing to downward motion, there is a rise in the mean skin-friction. Thus for $G = -5$, $E = -0{\cdot}01$, there is 1·7 per cent fall in the value of the mean skin-friction when the velocity of the plate in the upward direction is raised from 0 to 1 and there is 1·7 per cent rise when the velocity of the plate, moving in downward direction, is raised from 0 to 1. Owing to greater heating of the plate by the free convection currents, there is a fall in the value of the mean skin-friction for both upward and downward motion of the plate. Quantitatively, there is 73·2 per cent fall in the value of the mean skin-friction for air, when $V = 0$, 66 per cent fall for $V = 1$ and 89 per cent fall when $V = -1$. Hence we conclude that owing to greater heating of the plate by the free convection currents, the rate of fall in the mean skin-friction is more when the plate is moving in the downward direction.

However, owing to greater viscous dissipative heat, there is always a rise in the value of the mean skin-friction.

For water, when the plate is being heated by the free convection currents, there is a decrease in the value of the mean skin-friction when the plate moves upwards whereas there is an increase in the mean skin-friction when the plate moves downwards. Thus for possible experimental verification, we observe that for $G = -5$, $E = -0{\cdot}01$, there is 336 per cent decrease in the value of the mean skin-friction when V is changed from 0 to 1 for upward motion and 343 per cent rise in the value of the mean skin-friction when V is changed from 0 to -1 for downward motion. Greater heating of the plate by the free convection currents leads to a decrease in the value of the mean skin-friction for both upward and downward motion. Thus, for $E = -0{\cdot}01$ and the heating of the plate is such that G is increased from 5 to 10, there is 230 per cent fall in the value of the mean skin-friction for $V = 0$, 100 per cent fall for $V = 1$ and 48·8 per cent fall for $V = -1$. This helps us to conclude that in case of stationary plate, the rate of decrease in the value of the mean skin-friction is maximum whereas, compared to downward motion, the rate of decrease is more in case of upward motion.

Owing to greater viscous dissipative heat, there is always a rise in the value of the mean skin-friction for $G < 0$.

The rate of heat transfer in non-dimensional form is given by:

$$q = \frac{q'v}{Kv_0(T'_w - T'_\infty)} = -\left.\frac{d\theta}{dy}\right|_{y=0} = -\left.\frac{d\theta_0}{dy}\right|_{y=0} - \varepsilon\, e^{i\omega t}\left.\frac{d\theta_1}{dy}\right|_{y=0}. \tag{9}$$

Then the mean rate of heat transfer q_m is given by:

$$q_m = -\left.\frac{d\theta_0}{dy}\right|_{y=0} = P - PE\left[\frac{A^2}{2} - \frac{2GA}{P^2-1} + \frac{B^2P}{2}\right] \tag{10}$$

q_m is shown on Fig. 4(c, d).

We observe from this figure that, for air, when the plate is being cooled by the free convection currents, q_m increases when the plate is in upward motion and decreases when the plate is in downward motion. Quantitatively, for $G = 5$, $E = 0{\cdot}01$, when the velocity V changes from 0 to 1, in the upward direction, there is

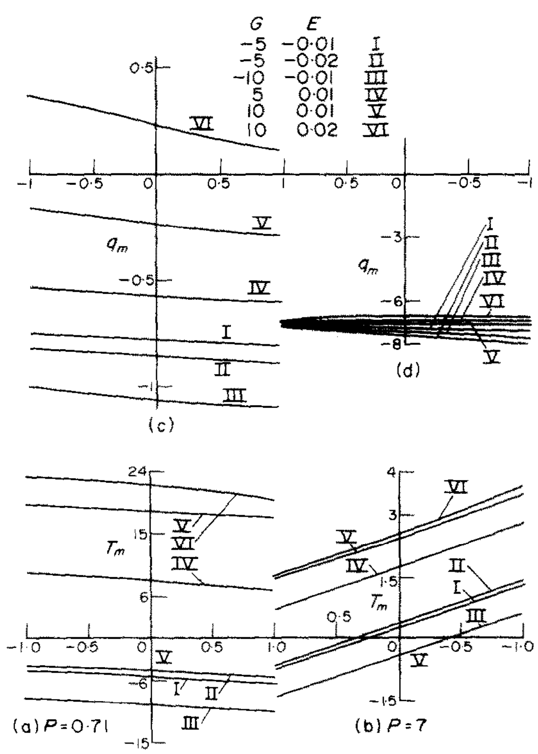

FIG. 4. Mean skin friction and mean rate of heat transfer.

5·2 per cent rise in the value of q_m whereas when the velocity changes from 0 to -1 in the downward direction, there is 7 per cent decrease in the value of q_m. Owing to greater cooling of the plate, there is again a fall in the value of q_m. For possible experimental verification, we observe that for $E = 0{\cdot}01$, and the plate is cooled more such that G is increased from 5 to 10, there is 59·6 per cent fall in the value of q_m for $V = 0$, 51·6 per cent fall for $V = 1$ and 69·8 per cent fall when $V = -1$. This leads us to conclude that the rate of fall in q_m due to greater cooling of the plate is more when the plate is moving in the downward direction.

Greater viscous dissipative heat causes a decrease in q_m for both upward and downward motion of the plate.

To understand the effects of the heating of the plate on q_m, we observe that an upward motion of the plate causes a rise in the value of q_m and the downward motion causes a fall in q_m, in case of air. Thus quantitatively, for $G = -5$, $E = -0{\cdot}01$, there is 3·8 per cent rise in q_m when V is changed from 0 to 1 whereas when V is changed from 0 to -1, there is 2·5 per cent fall in the value of q_m. But greater heating of the plate by the free convection currents causes a rise in the value of q_m. For experimental verification, for $E = -0{\cdot}01$, and the plate being heated such that G is increased from 5 to 10, there is 35·8 per cent rise in the value of q_m when $V = 0$, 38·2 per cent rise when $V = 1$ and 32·8 per cent when $V = -1$. This shows that the rate of increase in the value of the mean rate of heat transfer is more due to more heating of the plate which is moving in the upward direction.

In case of water, when the plate is being heated by the free convection currents, q_m increases when the plate is moving in the upward direction and decreases when the plate is moving in the downward direction. Greater heating of the plate causes negligible difference in the value of q_m. When the plate is being cooled by the free convection currents, q_m decreases when the plate is moving in the upward direction and increases when it is moving in the downward direction. There is negligible change in the value of q_m owing to greater cooling of the plate. Knowing the mean velocity and the temperature field, we now study the transient velocity. Following the procedure of [1,2], the expressions for the transient velocity can be derived. To save space, it is not shown here. The transient velocity profiles are shown on Fig. 5 for $\omega = 10$.

As in case of the flow past a stationary vertical plate, there is still a non-reverse type of flow of air and water when the plate is cooled externally and also moving in the upward or downward direction. However for $V \lessgtr 0$, there are reverse type of transient velocity profiles in case of air whereas they are non-reverse type for water. In [1], it was observed that there is 116 per cent rise in the maximum transient velocity of air owing to greater cooling of the plate such that G is increased from 5 to 10 and $V = 0$. Under similar circumstances and for $V = 1$, there is observed to be 111·7 per cent rise in the transient velocity. This leads us to conclude that the rate of increase in the magnitude of the maximum transient velocity, due to greater cooling of the plate, falls as the

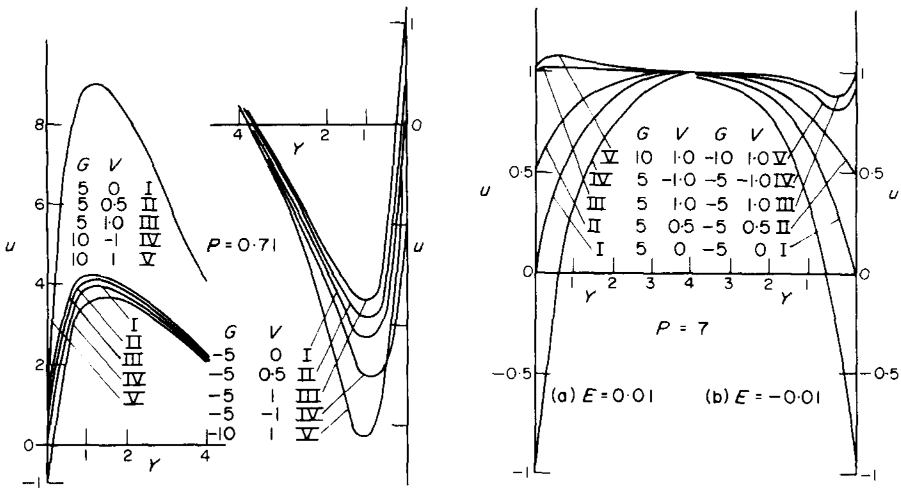

FIG. 5. Transient velocity profiles, $\omega = 10$; $\omega = \pi/2$; $\varepsilon = 0{\cdot}2$.

plate moves upward. In case of an externally heating of the plate, there is observed to be a rise in the transient velocity of air due to the upward motion of the plate and for $V > 0$, greater heating of the plate causes a fall in the transient velocity of air. For possible experimental verification, we observe that for $V = 1$ and G is increased from 5 to 10, due to greater heating of the plate, there is a 77·2 per cent fall in the maximum transient velocity whereas in [1], it was observed that for $V = 0$, there is 62·7 per cent drop in the maximum transient velocity under similar conditions of the greater heating of the plate. We conclude that owing to more heating of the plate by the free convection currents, the rate of fall in the maximum transient velocity is more when the plate is moving upward. Also, due to downward motion of the plate, there is again a fall in the transient velocity of air. The transient velocity of water also rises with the motion in the upward direction when the plate is either cooled or heated by the free convection currents. However, owing to greater cooling of the plate, moving in the upward direction, the transient velocity of water is observed to rise near the plate. Under similar circumstances, greater heating of the plate by the free convection currents, the transient velocity is observed to decrease near the plate and away from the plate, it rises. Thus for $V > 0$, the transient velocity of water near the plate behaves in a quite interesting and opposite manner due to greater cooling or heating of the plate.

On Fig. 6, the transient temperature profiles for air and water are shown for $\omega = 10$. We observe from this figure that when the plate is moving upward there is a fall in the transient temperature of air, water for both $G \gtrless 0$. But for $V < 0$, there is a rise in the transient temperature of air and water for $G \gtrless 0$. To understand the effects of greater heating or cooling of the plate, we observe that for $V > 0$, there is a rise in the transient temperature of air and a fall in the transient temperature of water owing to greater cooling of the plate. Again for $V > 0$, due to greater heating of the plate by the free convection currents, there is a fall in the transient temperature of air and a rise in the transient temperature of water near the plate. Thus the effects of the greater heating or cooling of the plate by the free

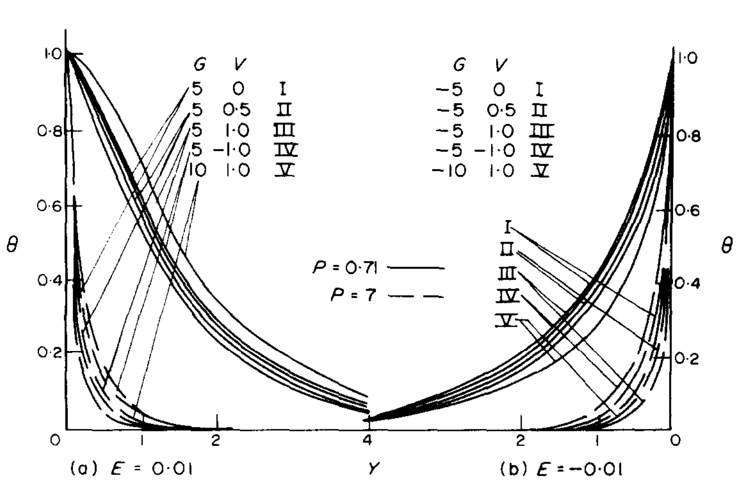

FIG. 6. Transient temperature profiles.

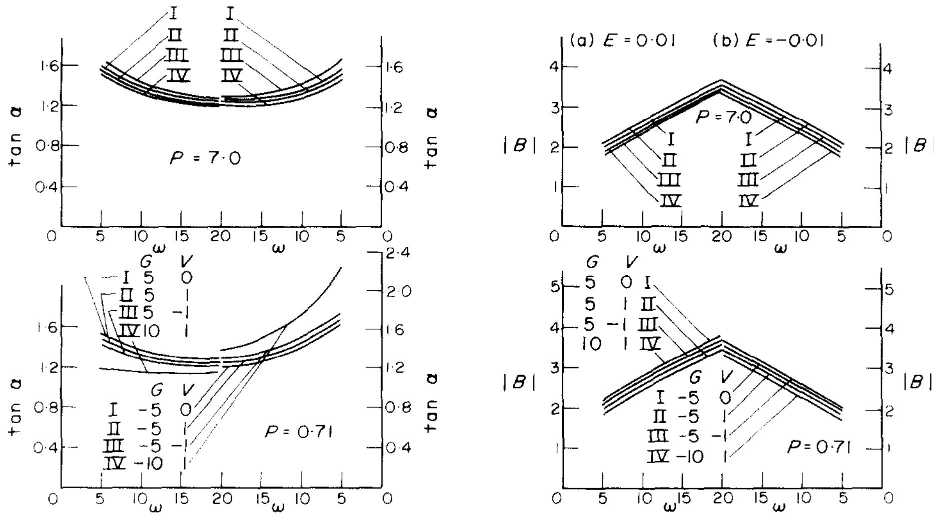

FIG. 7. Phase and amplitude of skin friction.

convection currents on the transient temperature depends upon the Prandtl number of the fluid. Following as in [1, 2], the amplitude and the phase of the skin-friction and the rate of heat transfer are calculated and are shown on Figs. 7 and 8 respectively.

On comparing the values of $|B|$ for air, with those for $V = 0$, we observe that there is a decrease in $|B|$ due to an upward motion of the plate and an increase in $|B|$ due to a downward motion of the plate. For $V > 0$, due to greater cooling of the plate, there is also a decrease in $|B|$ for air. For possible experimental verification, we observe that when the velocity of the plate is changed from 0 to 1, for $G = 5$, $E = 0{\cdot}01$, $\omega = 10$, there is a 0·38 per cent decrease in the value of $|B|$ for air. Again for $V = 1$, $E = 0{\cdot}01$, $\omega = 10$, owing to greater cooling of the plate such that G is increased from 5 to 10, there is a 1·9 per cent increase in the value of $|B|$ for air. But it was observed in [1], that for $V = 0$ and G increased from 5 to 10, there is a 16 per cent decrease in the value of $|B|$ for air when $E = 0{\cdot}01$ and $\omega = 10$. We conclude from this that the nature of the amplitude of the skin-friction is completely changed due to the upward motion of the plate. Again for $G > 0$, when the plate is moving in the downward direction, there is observed to be a rise in $|B|$ for air, when compared with those for $V = 0$. Thus, for $G > 0$, an upward or a downward motion affects the amplitude $|B|$ in quite the opposite manner.

In case of the heating of the plate externally, the effects of an upward or downward motion on $|B|$ are the same in nature as in case of cooling of the plate externally. However, greater heating of the plate, for $V = 1$, leads to a decrease in $|B|$. Thus for possible experimental verification, we observe that for $E = -0{\cdot}01$, $V = 1$, $\omega = 10$, there is a 1·1 per cent fall in the value of $|B|$ when G is increased from 5 to 10 owing to greater heating of the plate. Under similar circumstances, for $V = 0$, it has been observed that [1], there is a 10 per cent increase in $|B|$. This leads us to conclude that in the presence of the greater heating of the plate by the free convection currents, the nature of $|B|$ is completely changed due to an upward motion of the plate.

However in case of water, we observe that for $G > 0$, when the plate moves upward from a stationary state, there is observed to be a rise in $|B|$ and when the plate moves downward from the stationary state, there is a fall in the value of $|B|$. When $V = 1$, greater cooling of the plate leads to a rise in $|B|$. Thus the behaviour of $|B|$ in case of air and water, for $G > 0$, is different when the plate is moving in the upward or downward direction. But $|B|$ for air and water behaves in the same way, for $V > 0$, when the plate is being cooled more and more by the free convection currents. For $G < 0$, when the plate moves upward from a stationary state, there is observed to be a rise in $|B|$ for water and a fall in $|B|$ when the plate moves downward from a stationary state. This phenomenon is completely opposite from that in case of air. For $V = 1$, greater heating of the plate leads to a fall in the value of $|B|$ for water which has been observed also in case of air.

Figure 8 shows that when the plate is being cooled by the free convection currents, an upward motion of the plate, from rest, causes a fall in the amplitude $|Q|$ for air whereas a downward motion of the plate, from rest, causes a rise in the value of $|Q|$ for air. However for $V = 1$, greater cooling of the plate causes a rise in $|Q|$ for air. For possible experimental verification, we observe that for $V = 1$, $E = 0{\cdot}01$, $\omega = 10$ and G increased from 5 to 10 due to greater cooling of the plate, there is observed to be 125 per cent rise in the value of $|Q|$ for air. When the plate is being heated by the free convection currents, an upward motion of the plate, from rest, causes a rise in the value of $|Q|$ whereas a downward motion of the plate, from rest, causes a fall in the value of $|Q|$. But for $V = 1$, greater heating of the plate causes a rise in the value of $|Q|$ for air. Hence we conclude that greater heating or cooling of the plate moving in the upward direction always leads to a rise in the value of $|Q|$. Otherwise, for constant value of G,

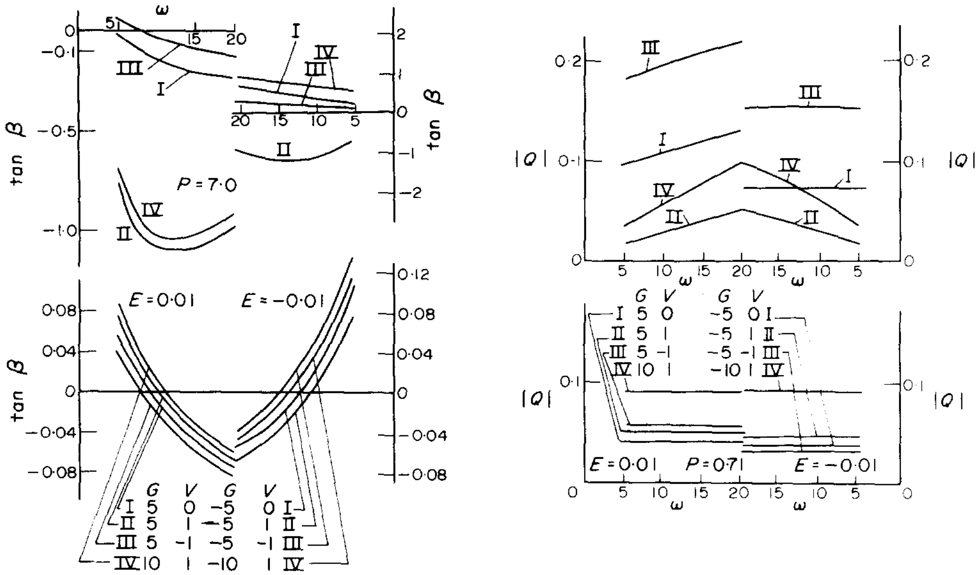

FIG. 8. Phase and amplitude of rate of heat transfer.

the nature of $|Q|$ depends upon the direction of the motion of the plate. In case of water, the behaviour of $|Q|$ for $G > 0$ is similar to that in case of air. But for $G < 0$, an upward motion of the plate from rest, causes a fall in the value of $|Q|$ and there is a rise in the value of $|Q|$ due to downward motion of the plate from rest. Greater heating of the plate in upward motion has the same effect as that in case of air, viz. a rise in the value of $|Q|$.

CONCLUSIONS

We now summarize some important observations:

Air

(1) Owing to greater cooling of the plate, there is a fall in the rate of increase in the maximum mean velocity in case of the upward motion of the plate and a rise in the rate of increase in the maximum mean velocity in case of the downward motion of the plate.

(2) For $G > 0$, the rate of increase in the maximum mean velocity is more for downward motion of the plate as compared to that for upward motion of the plate.

(3) There is always a fall in the rate of decrease in the maximum mean velocity owing to greater heating of the plate by the free convection currents for upward and downward motion of the plate.

(4) The rate of decrease in the maximum mean velocity is more in case of the plate moving in the upward direction than that in case of the motion of the plate in the downward direction.

(5) For $G \gtrless 0$, there is a rise in the mean temperature of air when the plate moves downwards and a fall in the mean temperature when the plate moves upwards.

(6) Greater cooling of the plate causes a rise in the mean temperature when the plate is moving in upward or downward direction.

(7) Greater heating of the plate causes a fall in the mean temperature for both upward and downward motion of the plate.

(8) For $G > 0$, an increase in the velocity of the plate in the upward direction leads to a fall in the mean temperature and that in the downward direction leads to an increase in the mean temperature of air. But for $G < 0$, an increase in the velocity of the plate in the upward or downward direction leads to a fall in the mean temperature of air.

(9) For $G \gtrless 0$, an upward motion of the plate causes a fall in the value of τ_m and the downward motion causes a rise in the value of τ_m.

(10) Owing to greater cooling of the plate, the rate of increase in the mean skin-friction is more when the plate is moving in the upward direction than that when it is moving in the downward direction.

(11) Due to greater heating of the plate, the rate of fall in the mean skin-friction is more when the plate is moving in the downward direction than that when it is moving in the upward direction.

(12) q_m increases in the upward motion and decreases in the downward motion for both $G \gtrless 0$.

(13) Owing to greater cooling of the plate, the rate of fall in q_m is more when the plate is moving in the downward direction than that when it is moving in the upward direction.

(14) Due to greater heating of the plate the rate of increase in the value of q_m is more when the plate is moving in the upward direction than that when it is moving in the downward direction.

Water

(15) Owing to greater heating of the plate, there is a fall in the mean velocity of water for upward or downward motion of the plate.

(16) Greater cooling of the plate causes a rise in the mean velocity of water near the plate moving in the upward direction.

(17) For $G \gtrless 0$, there is a fall in the mean skin-friction when the plate is moving upward and a rise, when the plate is moving downward.

(18) Owing to greater cooling of the plate, there is always a rise in the mean skin-friction. The rate of increase is more in case of the plate moving upward than that in case of the downward motion.

(19) Due to greater heating of the plate there is always a fall in the mean skin-friction and the rate of decrease is more for the plate moving in the upward direction than that in the downward motion of the plate.

(20) For $G < 0$, q_m increases when the plate moves in the upward direction and decreases when it moves in the downward direction. But for $G > 0$, q_m decreases in the case of upward motion of the plate and increases when there is a motion in the downward direction.

(21) There is negilgible effect of greater heating or cooling of the plate by the free convection currents, on q_m.

(22) The rate of increase in the magnitude of the maximum transient velocity of air, due to greater cooling of the plate, falls as the plate moves in the upward direction.

(23) Owing to greater heating of the plate by the free convection currents, the rate of fall in the maximum transient velocity for air, is more when the plate is moving in the upward direction.

(24) In water, for $G \gtrless 0$, the transient velocity rises due to the plate moving in the upward direction. For $V > 0$, owing to greater cooling of the plate, the transient velocity of water is observed to rise near the plate and due to greater heating of the plate, it falls near the plate.

(25) For air and water, when $G \gtrless 0$, there is a rise in the transient temperature due to the motion of the plate in the downward direction and a fall due to the motion in the upward direction.

(26) For $V > 0$, there is a rise in the transient temperature of air and a fall in the transient temperature of water owing to greater cooling of the plate whereas due to greater heating of the plate, the transient temperature falls in case of air and rises in case of water.

(27) For $G > 0$, when the plate moves upward from rest, $|B|$ for air, decreases and it increases when the plate is moving in the downward direction. For air, greater cooling of the plate causes a fall in the value of $|B|$ when the plate is moving in the upward direction.

(28) For $G < 0$, $|B|$ for air, behaves in the same manner as for $G > 0$ when the plate is moving in the upward or downward direction. But for $V > 0$, greater heating of the plate causes a decrease in $|B|$ for air.

(29) For water, when $G > 0$, there is a rise in $|B|$ when the plate starts moving in the upward direction and a fall in $|B|$ when it starts moving in the downward direction. Greater cooling of the plate causes a rise in $|B|$ for $V > 0$. For $G < 0$, there is a rise in $|B|$ when the plate starts moving upward and a fall in $|B|$ when it starts moving downward.

(30) For air and water, $\tan \alpha$ being positive for $V \gtrless 0$, there is always a phase lead.

(31) For air, the amplitude of the rate of heat transfer $|Q|$ increases when the plate starts moving downward and it decreases when it starts moving upward when $G > 0$. But for $V > 0$, greater cooling or heating of the plate causes a rise in $|Q|$. For $G < 0$, $|Q|$ increases when the plate starts moving upward and $|Q|$ decreases when the plate starts moving downward.

(32) For water, $|Q|$ behaves in the same manner as in case of air, when the plate starts moving upward and $G > 0$. But for $G < 0$, there is a fall in $|Q|$ when the plate starts moving upward and a rise in $|Q|$ when it starts moving downwards.

(33) For air, at large values of ω there is a phase-lag for $V \gtrless 0$ and $G \gtrless 0$ as $\tan \beta$ is always negative.

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INFLUENCE DE LA CONVECTION LIBRE SUR L'ECOULEMENT OSCILLATOIRE D'UN FLUIDE VISQUEUX INCOMPRESSIBLE SUR UNE PLAQUE VERTICALE EN MOUVEMENT UNIFORME AVEC ASPIRATION CONSTANTE

Résumé—On présente une étude analytique d'un écoulement bidimensionnel d'un fluide incompressible sur une plaque poreuse infinie dans les conditions suivantes: (i) la vitesse d'aspiration normale à la plaque est constante, (ii) la vitesse du courant extérieur oscille dans le temps autour d'une valeur moyenne constante, (iii) la température de la plaque est constante, (iv) il existe des courants de convection naturelle dûs à la différence de température entre la plaque et le fluide extérieur, et (v) la plaque est en mouvement uniforme ascendant ou descendant. La résolution approchée des équations non-linéaires couplées à permis d'obtenir des résultats présentées sous forme de graphiques pour la vitesse et la température moyennes, le frottement pariétal moyen, le flux de chaleur moyen, la vitesse et la température transitoires, l'amplitude et la phase du frottement pariétal et le flux de chaleur à la paroi. Dans le cas de l'air et de l'eau, on observe une diminution du frottement pariétal moyen lorsque la plaque, étant refroidie par les courants de convection libre, est en mouvement ascendant et une augmentation lorsqu'elle est en mouvement descendant. Dans le cas d'une plaque chauffée par les courants de convection libre, on constate une chute du frottement pariétal moyen pour un mouvement ascendant et une augmentation du frottement pariétal moyen pour un mouvement descendant. Les résultats sont présentés de manière quantitative au cours de la discussion.

AUSWIRKUNGEN DER FREIEN KONVEKTION AUF OSZILLIERENDE STRÖMUNG EINES ZÄHEN, INKOMPRESSIBLEN FLUIDS LÄNGS EINER SICH GLEICHMÄSSIG BEWEGENDEN VERTIKALEN PLATTE MIT KONSTANTER ABSAUGUNG

Zusammenfassung—Es wird die Berechnung der zweidimensionalen Strömung eines inkompressiblen, zähen Fluids längs einer unbegrenzten, porösen Platte gezeigt, wobei folgende Annahmen gelten:

1. Die Sauggeschwindigkeit normal zur Platte ist konstant.
2. Die Geschwindigkeit in der freien Strömung schwankt zeitlich um einen konstanten Mittelwert.
3. Die Plattentemperatur ist konstant.
4. Vorhandensein von freier Konvektionsströmung durch den Temperaturunterschied zwischen Plattentemperatur und Temperatur der freien Strömung.
5. Die Platte bewegt sich stetig nach oben oder nach unten.

Über einen näherungsweisen Lösungsweg für die gekoppelten, nicht-linearen Gleichungen erhält man folgende Ergebnisse, die grafisch dargestellt werden: mittlere Geschwindigkeit und Temperatur, mittlere Reibung, mittlere Größe der Wärmeübertragung, momentane Geschwindigkeit und Temperatur sowie Amplitude und Phase der Reibung und Größe der Wärmeübertragung.

Es wird festgestellt, daß für Luft und Wasser die mittlere Reibung abnimmt, wenn die durch freie Konvektionsströmung gekühlte Platte sich nach oben bewegt und zunimmt, wenn sie sich nach unten bewegt. Für die durch freie Konvektionsströmung beheizte Platte ergibt sich eine Abnahme der mittleren Reibung bei Aufwärtsbewegung und eine Zunahme bei Abwärtsbewegung. Quantitative Ergebnisse werden im Rahmen der Diskussion dargeboten.

ВЛИЯНИЕ СВОБОДНОЙ КОНВЕКЦИИ НА КОЛЕБАТЕЛЬНЫЙ ПОТОК ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ, ОБТЕКАЮЩЕЙ РАВНОМЕРНО ДВИЖУЩУЮСЯ ВЕРТИКАЛЬНУЮ ПЛАСТИНУ ПРИ ПОСТОЯННОМ ОТСОСЕ

Аннотация — Представлен анализ двумерного потока несжимаемой вязкой жидкости, которая обтекает неограниченную пористую пластину при условиях: (1) скорость отсоса по нормали к пластине постоянна; (2) скорость свободного потока изменяется во времени, достигая среднего значения; (3) температура пластины постоянна; (4) наличие потоков при свободной конвекции вследствие разности температур между температурой пластины и температурой свободного потока; (5) пластина перемещается постоянно вверх и вниз. Решая связанные нелинейные уравнения с помощью приближенного метода, графически изображены результаты по средней скорости и температуре, среднему поверхностному трению, средней скорости теплообмена, по нестационарной скорости и температуре, по амплитуде и фазе поверхностного трения и по скорости теплообмена. Замечено, что среднее поверхностное трение для воздуха и воды уменьшается, когда пластина, охлаждаемая свободноконвективным потоком, движется вверх и возрастает, когда пластины движется вниз. Когда, пластина нагрета свободноконвективным потоком, наблюдается падение среднего поверхностного трения в случае восходящего движения, а случае же нисходящего движения среднее поверхностное трение возрастает. Представлены количественные результаты.